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(i)の形の数列として、

(1) $a_n = an + b = a \times n + b \times 1$　(2) $a_n = an^2 + b = a \times n \times n + b$　(3) $a_n = an + \dfrac{b}{n} = a \times n + b \div n$

(ii)の形の数列として、

(4) $a_n = an^n + b = n^n \times a + b$　(5) $a_n = 2^n a + b = 2^n \times a + b$　(6) $a_n = 3^n a + b = 3^n \times a + b$

を、それぞれ考える。

$a_1 = 2,\ a_2 = 3$ となるように、定数 $a,\ b$ を定めると

(1) $a + b = 2,\ 2a + b = 3$　$\therefore a = b = 1$　(2) $a + b = 2,\ 4a + b = 3$　$\therefore a = \dfrac{1}{3},\ b = \dfrac{5}{3}$

(3) $a + b = 2,\ 2a + \dfrac{b}{2} = 3$　$\therefore a = \dfrac{4}{3},\ b = \dfrac{2}{3}$　(4) $a + b = 2,\ 4a + b = 3$　$\therefore a = \dfrac{1}{3},\ b = \dfrac{5}{3}$

(5) $2a + b = 2,\ 4a + b = 3$　$\therefore a = \dfrac{1}{2},\ b = 1$　(6) $3a + b = 2,\ 9a + b = 3$　$\therefore a = \dfrac{1}{6},\ b = \dfrac{3}{2}$

以上により

$a_n = n + 1,\ \dfrac{1}{3}n^2 + \dfrac{5}{3},\ \dfrac{4}{3}n + \dfrac{2}{3n},\ \dfrac{1}{3}n^n + \dfrac{5}{3},\ 2^{n-1} + 1,\ \dfrac{1}{2} \cdot 3^{n-1} + \dfrac{3}{2}$　……(答)