

2002 年京大文 1

$$S_{n+1} - S_n = n^2 a_{n+1} - (n-1)^2 a_n = a_{n+1} \quad (n^2 - 1)a_{n+1} = (n-1)^2 a_n$$

$$n \geq 2 \text{ のとき } (n+1)a_{n+1} = (n-1)a_n \quad \therefore (n+1)na_{n+1} = n(n-1)a_n$$

$$n(n-1)a_n \text{ は一定であるから } n(n-1)a_n = 2a_2 = 2 \quad \therefore a_n = \frac{2}{n(n-1)} = \frac{2}{n-1} - \frac{2}{n} \quad (n \geq 2)$$

$$S_n = 2 \left\{ \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right) \right\} = 2 \left(1 - \frac{1}{n}\right) = (n-1)^2 a_n \text{ であるから、確かに成立。}$$

$$\therefore a_1 = 0, a_n = \frac{2}{n(n-1)} \quad (n \geq 2) \quad \cdots \cdots \text{(答)}$$