$$\int_{10}^{100} \log_{10} x \, dx = \frac{1}{\log 10} \int_{10}^{100} \log x \, dx = \frac{1}{\log 10} \Big[x \log x - x \Big]_{10}^{100} = \frac{1}{\log 10} (100 \log 100 - 100 - 10 \log 10 + 10)$$

$$= \frac{1}{\log 10} (190 \log 10 - 90) = 190 - \frac{90}{\log 10} = 190 - 90 \log_{10} e$$

$0.434 < \log_{10} e < 0.435$ より $39.06 < 90 \log_{10} e < 39.15$ $150.85 < 190 - 90 \log_{10} e < 150.94$

$n < \int_{10}^{100} \log_{10} x \, dx$ を満たす最大の自然数 n は $\therefore n = 150$ ……(答)