2021 年京大文 2

$x^2-\frac{1}{2}x-\frac{1}{2}=\left(x+\frac{1}{2}\right)(x-1)$であるから

$-\frac{1}{2}<x<1$ のとき　$x^2-\frac{1}{2}x-\frac{1}{2}<0$　$x\leqq-\frac{1}{2},1\leqq x$のとき　$x^2-\frac{1}{2}x-\frac{1}{2}\geqq 0$

$$\int_{-1}^{1}\left|x^2-\frac{1}{2}x-\frac{1}{2}\right|dx=\int_{-1}^{-\frac{1}{2}}\left(x+\frac{1}{2}\right)(x-1)dx-\int_{-\frac{1}{2}}^{1}\left(x+\frac{1}{2}\right)(x-1)dx$$

$$=\int_{-\frac{1}{2}}^{0}t\left(t-\frac{3}{2}\right)dt-\int_{0}^{\frac{3}{2}}t\left(t-\frac{3}{2}\right)dt=\left[\frac{t^3}{3}-\frac{3}{4}t^2\right]_{-\frac{1}{2}}^{0}-\left[\frac{t^3}{3}-\frac{3}{4}t^2\right]_{0}^{\frac{3}{2}}$$

※$t=x+\frac{1}{2}$と置換

$$=\frac{1}{24}+\frac{3}{16}-\left(\frac{9}{8}-\frac{27}{16}\right)=\frac{2+9-54+81}{48}=\frac{19}{24}$$ ……(答)