2023年京大文 4

漸化式の分母を払うと　$na_n = S_n + n(n-1)\cdot 2^n$ ――①

①より　$(n+1)a_{n+1} = S_{n+1} + (n+1)n\cdot 2^{n+1}$ ――②

②−①より　$(n+1)a_{n+1} - na_n = a_{n+1} + (n+1)n\cdot 2^{n+1} - n(n-1)\cdot 2^n$

$n(a_{n+1}-a_n) = (2n^2+2n-n^2+n)\cdot 2^n = n(n+3)\cdot 2^n$　$\therefore a_{n+1}-a_n = (n+3)\cdot 2^n$

$n \geqq 2$ のとき　$\displaystyle\sum_{k=1}^{n-1}(a_{k+1}-a_k) = a_n - a_1 = \sum_{k=1}^{n-1}\{(k+3)\cdot 2^k\}$　$\displaystyle\therefore a_n = \sum_{k=1}^{n-1}\{(k+3)\cdot 2^k\} + 3$

$n \geqq 2$ のとき　$\displaystyle T_n = \sum_{k=1}^{n-1}\{(k+3)\cdot 2^k\}$ とすると

$$T_n = 4\cdot 2^1 + 5\cdot 2^2 + 6\cdot 2^3 + \cdots + (n+1)\cdot 2^{n-2} + (n+2)\cdot 2^{n-1}$$

$$2T_n = \qquad 4\cdot 2^2 + 5\cdot 2^3 + \cdots + \qquad n\cdot 2^{n-2} + (n+1)\cdot 2^{n-1} + (n+2)\cdot 2^n$$

辺々引くと

$$-T_n = 4\cdot 2^1 + 2^2 + 2^3 + \cdots + 2^{n-2} + 2^{n-1} - (n+2)\cdot 2^n$$

$$T_n = (n+2)\cdot 2^n - (2^1 + 2^2 + 2^3 + \cdots + 2^{n-2} + 2^{n-1}) - 6 = (n+2)\cdot 2^n - \frac{2(2^{n-1}-1)}{2-1} - 6$$

$$= (n+2)\cdot 2^n - (2^n - 2) - 6 = (n+1)\cdot 2^n - 4$$

$$\therefore a_n = (n+1)\cdot 2^n - 1$$

$a_1 = 3$ より、$n = 1$ でも成立するから、求める一般項は　$\therefore a_n = (n+1)\cdot 2^n - 1$ ……(答)