1968 年京大理 $\boxed{6}$

$$\frac{1}{x}\int_0^x \frac{dt}{\sqrt{1-t}} = \frac{1}{x}\Big[-2\sqrt{1-t}\Big]_0^x = \frac{2}{x}(1-\sqrt{1-x}) = \frac{1}{\sqrt{1-y}}$$

$$\sqrt{1-y} = \frac{x}{2(1-\sqrt{1-x})} = \frac{x(1+\sqrt{1-x})}{2x} = \frac{1}{2}(1+\sqrt{1-x})$$

$$1-y = \frac{1}{4}(1+\sqrt{1-x})^2 = \frac{1}{4}(2-x+2\sqrt{1-x}) = \frac{1}{2}-\frac{1}{4}x+\frac{1}{2}\sqrt{1-x}$$

$$\therefore y = \frac{1}{2}+\frac{1}{4}x-\frac{1}{2}\sqrt{1-x} \qquad \therefore \frac{dy}{dx} = \frac{1}{4}+\frac{1}{4\sqrt{1-x}} > 0$$

したがって、y は x の増加関数であり、$\lim_{x\to 1} y = \frac{3}{4}$ ……(答)