

1973 年京大文 [6]

$$y = ax^2 - 4 \text{ と } y = 5 \text{ の交点の } x \text{ 座標は } ax^2 = 9 \quad x^2 = \frac{9}{a} \quad x = \pm \frac{3}{\sqrt{a}}$$

右図より

$$\begin{aligned} S &= 2 \int_0^{\frac{3}{\sqrt{a}}} \{5 - (ax^2 - 4)\} dx = 2 \int_0^{\frac{3}{\sqrt{a}}} (9 - ax^2) dx = 2 \left[9x - \frac{a}{3} x^3 \right]_0^{\frac{3}{\sqrt{a}}} \\ &= 2 \left(\frac{27}{\sqrt{a}} - \frac{a}{3} \cdot \frac{27}{a\sqrt{a}} \right) = \frac{36}{\sqrt{a}} \quad \dots\dots (\text{答}) \end{aligned}$$

$$V = \pi \int_{-4}^5 x^2 dy = \pi \int_{-4}^5 \frac{y+4}{a} dy = \pi \left[\frac{(y+4)^2}{2a} \right]_{-4}^5 = \frac{81}{2a} \pi \quad \dots\dots (\text{答})$$

$$\frac{1}{\sqrt{a}} = \frac{S}{36} \text{ であるから } V = \frac{81}{2} \pi \cdot \frac{S^2}{36^2} = \frac{\pi}{32} S^2 \quad \therefore V = \frac{\pi}{32} S^2 \quad \dots\dots (\text{答})$$

