1994 年京大理 6

(1)

$x = 2\cos\theta - \cos 2\theta,\ y = 2\sin\theta - \sin 2\theta$ とすると

$$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta,\ \frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2 = 8 - 8(\cos 2\theta\cos\theta + \sin 2\theta\sin\theta) = 8 - 8\cos(2\theta - \theta) = 8(1 - \cos\theta) = 16\sin^2\frac{\theta}{2}$$

$$\therefore L = \int_0^{2\pi}\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\,d\theta = 4\int_0^{2\pi}\sin\frac{\theta}{2}\,d\theta = 4\left[-2\cos\frac{\theta}{2}\right]_0^{2\pi} = 16$$ ……(答)

(2)

$0 < \theta_n < 2\pi$ より

$$\frac{L}{n} = \frac{16}{n} = 4\int_0^{\theta_n}\sin\frac{\theta}{2}\,d\theta = 4\left[-2\cos\frac{\theta}{2}\right]_0^{\theta_n} = 8\left(1 - \cos\frac{\theta_n}{2}\right) = 16\sin^2\frac{\theta_n}{4}$$

$$n = \frac{1}{\sin^2\frac{\theta_n}{4}} \qquad \sqrt{n}\,\theta_n = \frac{\theta_n}{\sin\frac{\theta_n}{4}} = 4\cdot\frac{\frac{\theta_n}{4}}{\sin\frac{\theta_n}{4}}$$

$n \to \infty$ のとき、$\theta_n \to 0$ 、$\dfrac{\sin\frac{\theta_n}{4}}{\frac{\theta_n}{4}} \to 1$ であるから　$\therefore \lim_{n\to\infty}\sqrt{n}\,\theta_n = 4$ ……(答)