1999年京大後期文

\[ \sin^2(\alpha + \beta) - (\sin^2 \alpha + \sin^2 \beta) \]
\[ = (\sin \alpha \cos \beta + \cos \alpha \sin \beta)^2 - (\sin^2 \alpha + \sin^2 \beta) \]
\[ = \sin^2 \alpha \cos^2 \beta + 2 \sin \alpha \cos \alpha \sin \beta \cos \beta + \cos^2 \alpha \sin^2 \beta - (\sin^2 \alpha + \sin^2 \beta) \]
\[ = \sin^2 \alpha (\cos^2 \beta - 1) + 2 \sin \alpha \cos \alpha \sin \beta \cos \beta + \sin^2 \beta (\cos^2 \alpha - 1) \]
\[ = -\sin^2 \alpha \sin^2 \beta + 2 \sin \alpha \cos \alpha \sin \beta \cos \beta - \sin^2 \beta \sin^2 \alpha \]
\[ = 2 \sin \alpha \sin \beta (\cos \alpha \cos \beta - \sin \alpha \sin \beta) = 2 \sin \alpha \sin \beta \cos(\alpha + \beta) \]

\[ \alpha > 0^\circ, \beta > 0^\circ, \alpha + \beta < 180^\circ \]より、\( \sin^2 \alpha + \sin^2 \beta = \sin^2(\alpha + \beta) \)のとき \( \cos(\alpha + \beta) = 0 \) \( \therefore \alpha + \beta = 90^\circ \)

\[ 0^\circ < \alpha < 90^\circ \]として \( \sin \alpha + \sin \beta = \sin \alpha + \sin(90^\circ - \alpha) = \sin \alpha + \cos \alpha = \sqrt{2} \sin(\alpha + 45^\circ) \)

\[ 45^\circ < \alpha + 45^\circ < 135^\circ \]であるから \( \frac{1}{\sqrt{2}} < \sin(\alpha + 45^\circ) \leq 1 \)

求める範囲は \( \therefore 1 < \sin \alpha + \sin \beta \leq \sqrt{2} \) \( \cdots \)（答）等号成立は、\( \alpha = \beta = 45^\circ \)のとき。