2002 年京大後期理 6

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin(x-y)f(y)dy = \sin x\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y)\cos y dy - \cos x\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y)\sin y dy$$

$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y)\cos y dy,\ B=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(y)\sin y dy$ は定数であるから

$$f(x)+A\sin x - B\cos x = x+1 \quad \therefore f(x) = -A\sin x + B\cos x + x + 1$$

これを代入すると

$$A=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-A\sin y + B\cos y + y + 1)\cos y dy = -A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin y\cos y dy + B\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 y dy + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (y+1)\cos y dy$$

$$B=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-A\sin y + B\cos y + y + 1)\sin y dy = -A\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 y dy + B\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin y\cos y dy + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (y+1)\sin y dy$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin y\cos y dy = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2y dy = \frac{1}{2}\left[-\frac{1}{2}\cos 2y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 y dy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2y}{2} dy = \left[\frac{1}{2}y + \frac{1}{4}\sin 2y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^2 y + \cos^2 y)dy = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dy = \pi \text{ より} \quad \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 y dy = \pi - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 y dy = \frac{\pi}{2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (y+1)\cos y dy = \left[(y+1)\sin y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin y dy = \left(\frac{\pi}{2}+1\right) + \left(-\frac{\pi}{2}+1\right) + \left[\cos y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (y+1)\sin y dy = \left[-(y+1)\cos y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos y dy = \left[\sin y\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2$$

これより

$$A=\frac{\pi}{2}B+2,\ B=-\frac{\pi}{2}A+2 \quad \therefore A=\frac{4(\pi+2)}{\pi^2+4},\ B=-\frac{4(\pi-2)}{\pi^2+4}$$

$$\therefore f(x) = -\frac{4(\pi+2)}{\pi^2+4}\sin x - \frac{4(\pi-2)}{\pi^2+4}\cos x + x + 1 \quad \cdots\cdots(\text{答})$$