2015 年京大理 1

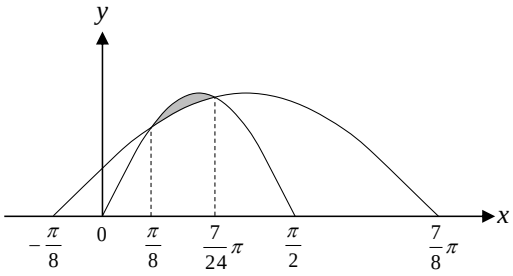
$\sin\left(x+\frac{\pi}{8}\right)=\sin 2x$ とすると　$\sin\left(x+\frac{\pi}{8}\right)-\sin 2x=0$　$2\cos\left\{\frac{1}{2}\left(3x+\frac{\pi}{8}\right)\right\}\sin\left\{\frac{1}{2}\left(\frac{\pi}{8}-x\right)\right\}=0$

$0\leqq x\leqq\frac{\pi}{2}$ のとき　$\frac{\pi}{8}\leqq 3x+\frac{\pi}{8}\leqq\frac{13}{8}\pi$　$-\frac{3}{8}\pi\leqq\frac{\pi}{8}-x\leqq\frac{\pi}{8}$

$3x+\frac{\pi}{8}=\pi$ または $\frac{\pi}{8}-x=0$ であるから　$\therefore x=\frac{\pi}{8},\ \frac{7}{24}\pi$

$\frac{\pi}{8}<\frac{\pi}{4}<\frac{7}{24}\pi<\frac{3}{8}\pi$ より、$0\leqq x\leqq\frac{\pi}{2}$ において、

$y=\sin\left(x+\frac{\pi}{8}\right)$ と $y=\sin 2x$ で囲まれる領域は、右図の通り。



求める体積は

$$\pi\int_{\frac{\pi}{8}}^{\frac{7}{24}\pi}\sin^2 2x\,dx-\pi\int_{\frac{\pi}{8}}^{\frac{7}{24}\pi}\sin^2\left(x+\frac{\pi}{8}\right)dx$$

$$=\frac{\pi}{2}\int_{\frac{\pi}{8}}^{\frac{7}{24}\pi}(1-\cos 4x)dx-\frac{\pi}{2}\int_{\frac{\pi}{8}}^{\frac{7}{24}\pi}\left\{1-\cos 2\left(x+\frac{\pi}{8}\right)\right\}dx=\frac{\pi}{2}\left[x-\frac{1}{4}\sin 4x\right]_{\frac{\pi}{8}}^{\frac{7}{24}\pi}-\frac{\pi}{2}\left[x-\frac{1}{2}\sin 2\left(x+\frac{\pi}{8}\right)\right]_{\frac{\pi}{8}}^{\frac{7}{24}\pi}$$

$$=\frac{\pi}{2}\left(\frac{7}{24}\pi+\frac{1}{8}-\frac{\pi}{8}+\frac{1}{4}\right)-\frac{\pi}{2}\left(\frac{7}{24}\pi-\frac{1}{4}-\frac{\pi}{8}+\frac{1}{2}\right)=\frac{\pi}{16}\quad\cdots\cdots(\text{答})$$