

2023 年京大文 [5]

$$\int_{-1}^1 (x-y)^2 f(y) dy = x^2 \int_{-1}^1 f(y) dy - 2x \int_{-1}^1 y f(y) dy + \int_{-1}^1 y^2 f(y) dy$$

$$A = \int_{-1}^1 f(y) dy, B = 2 \int_{-1}^1 y f(y) dy, C = \int_{-1}^1 y^2 f(y) dy \text{ は定数であるから}$$

$$f(x) = 2x^2 + x + \frac{5}{3} - (Ax^2 - Bx + C) = (2-A)x^2 + (1+B)x + \frac{5}{3} - C$$

$$A = \int_{-1}^1 \left\{ (2-A)y^2 + (1+B)y + \frac{5}{3} - C \right\} dy = 2 \left[(2-A) \frac{y^3}{3} + \left(\frac{5}{3} - C \right) y \right]_0^1 = 2 \left(\frac{2-A}{3} + \frac{5}{3} - C \right)$$

$$3A = 2(2-A+5-3C) \quad \therefore 5A+6C=14 \quad \text{---①}$$

$$B = 2 \int_{-1}^1 \left\{ (2-A)y^3 + (1+B)y^2 + \left(\frac{5}{3} - C \right) y \right\} dy = 4 \left[(1+B) \frac{y^3}{3} \right]_0^1 = \frac{4(1+B)}{3}$$

$$3B = 4(1+B) \quad \therefore B = -4$$

$$C = \int_{-1}^1 \left\{ (2-A)y^4 + (1+B)y^3 + \left(\frac{5}{3} - C \right) y^2 \right\} dy = 2 \left[(2-A) \frac{y^5}{5} + \left(\frac{5}{3} - C \right) \frac{y^3}{3} \right]_0^1 = 2 \left(\frac{2-A}{5} + \frac{5-3C}{9} \right)$$

$$45C = 2(18-9A+25-15C) \quad \therefore 18A+75C=86 \quad \text{---②}$$

$$\text{①、②より } A=2, C=\frac{2}{3}$$

$$f(x) = (2-A)x^2 + (1+B)x + \frac{5}{3} - C \text{ に代入すると } \therefore f(x) = -3x + 1 \quad \dots\dots (\text{答})$$