2026 年京大文 $\boxed{4}$

(1)

$1 \leqq k \leqq 2$ のとき　$0 \leqq \log_3 k < 1$　$\therefore [\log_3 k] = 0$

$3 \leqq k \leqq 8$ のとき　$1 \leqq \log_3 k < 2$　$\therefore [\log_3 k] = 1$

$9 \leqq k \leqq 26$ のとき　$2 \leqq \log_3 k < 3$　$\therefore [\log_3 k] = 2$

$\therefore a_{26} = 2 \times 0 + 6 \times 1 + 18 \times 2 = 42$ ……(答)

(2)

$3^{n-1} \leqq k \leqq 3^n - 1$ のとき　$n - 1 \leqq \log_3 k < n$　$\therefore [\log_3 k] = n - 1$

$[\log_3 k] = n - 1$ となるkの個数は、$(3^n - 1) - 3^{n-1} + 1 = 2 \cdot 3^{n-1}$であるから

$$a_m = 2 \times \{0 \cdot 3^0 + 1 \cdot 3^1 + 2 \cdot 3^2 + \cdots + (N-1) \cdot 3^{N-1}\}$$

$$3a_m = 2 \times \{0 \cdot 3^1 + 1 \cdot 3^2 + 2 \cdot 3^3 + \cdots + (N-2) \cdot 3^{N-1} + (N-1) \cdot 3^N\}$$

辺々引くと

$$-2a_m = 2 \times \{3^1 + 3^2 + \cdots + 3^{N-1} - (N-1) \cdot 3^N\}$$

$$\therefore a_m = (N-1) \cdot 3^N - (3^1 + 3^2 + \cdots + 3^{N-1}) = (N-1) \cdot 3^N - \frac{3(3^{N-1} - 1)}{3 - 1}$$

$$= (N-1) \cdot 3^N - \frac{1}{2} \cdot 3^N + \frac{3}{2} = \frac{(2N-3) \cdot 3^N + 3}{2}$$ ……(答)