2014 年東大文 $\boxed{1}$

(1)

$$f(x)=-2x^2+8tx-12x+t^3-17t^2+39t-18=-2\left\{x^2-(4t-6)x\right\}+t^3-17t^2+39t-18$$

$$=-2\{x-(2t-3)\}^2+2(2t-3)^2+t^3-17t^2+39t-18$$

$$=-2\{x-(2t-3)\}^2+t^3-9t^2+15t$$

したがって、$f(x)$ の最大値は　$\therefore t^3-9t^2+15t$ ……(答)

(2)

$$g(t)=t^3-9t^2+15t \qquad g'(t)=3t^2-18t+15=3(t^2-6t+5)=3(t-1)(t-5)$$

$g(t)$ の増減は右の通り。ここで、

t	$-\frac{1}{\sqrt{2}}$	$\cdots$	1	$\cdots$	5	$\cdots$
$f'(t)$		＋	0	－	0	＋
$f(t)$		↗		↘		↗

$$g\left(-\frac{1}{\sqrt{2}}\right)=-\frac{1}{2\sqrt{2}}-9\cdot\frac{1}{2}-\frac{15}{\sqrt{2}}=-\frac{9}{2}-\frac{31}{4}\sqrt{2}=-\frac{18+31\sqrt{2}}{4}$$

$$g(5)=125-225+75=-25$$

$$-\frac{18+31\sqrt{2}}{4}+25=\frac{82-31\sqrt{2}}{4}>\frac{82-31\cdot 2}{4}=5>0 \qquad \therefore -\frac{18+31\sqrt{2}}{4}>-25$$

したがって、$g(t)$ の最小値は　$\therefore -25$ ……(答)