2026 年東大文 1

$$y=k(x-\alpha)(\beta-x)=k\{-x^2+(\alpha+\beta)x-\alpha\beta\}=k\left\{-\left(x-\frac{\alpha+\beta}{2}\right)^2+\frac{(\alpha+\beta)^2}{4}-\alpha\beta\right\}$$

Cの頂点は$(-3,1)$であるから　$\dfrac{\alpha+\beta}{2}=-3$　$\therefore\alpha+\beta=-6$――①

$$k\left\{\frac{(\alpha+\beta)^2}{4}-\alpha\beta\right\}=k(9-\alpha\beta)=1\quad 9-\alpha\beta=\frac{1}{k}\qquad\therefore\alpha\beta=9-\frac{1}{k}$$ ――②

Cはy軸と $-2\leqq y\leqq 0$ の範囲で交わるから

$-2\leqq -k\alpha\beta\leqq 0$　$0\leqq k\alpha\beta\leqq 2$　②を代入して　$0\leqq 9k-1\leqq 2$　$\therefore\dfrac{1}{9}\leqq k\leqq\dfrac{1}{3}$――③

$$S=k\int_\alpha^\beta(x-\alpha)(\beta-x)dx=-k\int_\alpha^\beta(x-\alpha)(x-\beta)dx=\frac{k(\beta-\alpha)^3}{6}$$

①、②より　$(\beta-\alpha)^2=(\alpha+\beta)^2-4\alpha\beta=36-4\left(9-\dfrac{1}{k}\right)=\dfrac{4}{k}$　$\therefore S=\dfrac{k}{6}\left(\dfrac{4}{k}\right)^{\frac{3}{2}}=\dfrac{4}{3\sqrt{k}}$

③より　$\dfrac{1}{3}\leqq\sqrt{k}\leqq\dfrac{1}{\sqrt{3}}$　$\sqrt{3}\leqq\dfrac{1}{\sqrt{k}}\leqq 3$　$\therefore\dfrac{4}{3}\sqrt{3}\leqq S\leqq 4$ ……(答)