ARCS AND VALUATIONS

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In [7], Nash posed a problem: if the set of the families of arcs through the singularities on a variety (these families are called the Nash components) corresponds bijectively to the set of essential divisors of resolutions of the singularities. This problem is affirmatively answered for some 2-dimensional singularities by A. Reguera and M. Lejeune-Jalabert [8], [6], [9] and toric singularities of arbitrary dimension by S. Ishii and J. Kollár [5]. On the other hand this problem is negatively answered in general. The paper [5] gives a counter example of dimension greater than or equal to 4. Therefore the Nash problem should be changed to a problem to determine the divisors corresponding to the Nash components.

We can generalize this problem into the characterization problem for valuations corresponding to the irreducible components of contact loci $\mathrm{Cont}^{\geq m}(\mathfrak{a})$ which are introduced by L. Ein, R. Lazarsfeld and M. Mustaţă ([1]). In this talk we introduce the maximal divisorial set $C_X(v)$ in the arc space of a variety X corresponding to a divisorial valuation v. An irreducible component of a contact locus is a maximal divisorial set. In order to characterize the valuations corresponding to the irreducible components of a contact locus, it is essential to translate the inclusion relation between two maximal divisorial sets to a relation between the corresponding divisorial valuations. The most natural candidate for the translated relation is the value-inequality relation, i.e., $v(f) \leq v'(f)$ for every regular function f on the affine variety X. If the variety X and the valuations v, v' are toric, we have the equivalence: $v(f) \leq v'(f)$ for every regular function f on X if and only if $C_X(v) \supset C_X(v')$. We study this value-inequality relation. We describe a maximal divisorial set on the arc space of non-singular variety and determine the necessary and sufficient condition for the value-inequality relation. As a result we show that this most natural relation is not the translation of the inclusion relation of the maximal divisorial sets.

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